Dean Bank Primary and Nursery School



Progression In Written Methods Of Calculation

September 2019

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PROGRESSION IN CALCULATION

From Foundation Stage to Year 6

The aim for written calculations is different from the aim for mental calculations. With mental work, the aim is to teach children a repertoire of strategies from which to select. With written calculations the ultimate aim is proficiency in a compact method for each operation.

Mental calculation

Strategies for mental calculation are introduced from Y1 or Y2 to Y3 and developed further in Y4, Y5 and Y6. All children, apart from those with significant special educational needs, should be introduced to the full range of mental calculation strategies when they have the necessary pre-requisite skills. Children with significant special needs should learn a narrow range of strategies which are generally applicable.

Written calculations

Building on the mental strategies they have used so that they can understand the process, children need first to be taught to record their methods in an expanded form. When ready, they are taught how to refine the recording to make it more compact.

The methods and layouts to be taught for each operation are detailed in this document.

Challenges to teachers

- Ensuring that recall skills are established first so children can concentrate on a written method without reverting to first principles
- Making sure that, once written methods are introduced, children continue to look out for and recognise the special cases that can be done mentally;
- Catering for children who progress at different rates; some may grasp a compact method of calculation while others may never do so without considerable help;
- Catering for children who can carry out some standard methods successfully, e.g. for +, but not ;
- Recognising that children tend to forget a standard method if they have no understanding of what they are doing

Often the compactness of a vertical method conceals how mathematical principles are applied, e.g. children may use place value when working mentally, but be confused in written work because they do not understand how place value relates to 'carrying'. There can be long-lasting problems for those taught compact, vertical methods before they understand what they are doing.

Simply correcting children's errors may help in the short-term, but not permanently. They need to understand why a particular method works rather than simple following a set of rules. They can then fall back to a simpler method if uncertain or to check their answer.

NOTE: In the following guidance, suggestions are given as to when written methods and particular layouts should be introduced. However, the most importance thing to consider rather than children's age, is whether they have the necessary pre-requisite skills.

Progression towards a written method for addition

Before the introduction of formal written methods for addition, children should be able to:

- 1. recall all addition pairs to 9 + 9 and complements in 10;
- 2. add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- 3. add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- 4. partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

EARLY STAGES (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities.

They will begin to relate addition to combining two groups of objects, first by counting all and then by counting on from the largest number.

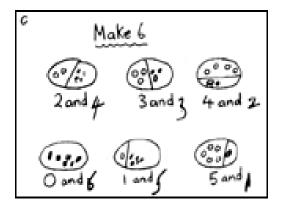
They will find one more than a given number.

In practical activities and through discussion they will begin to use the vocabulary involved in addition.



'You have five apples and I have three apples. How many apples are there altogether?'

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.



<u>YEAR 1</u>

Statutory requirements

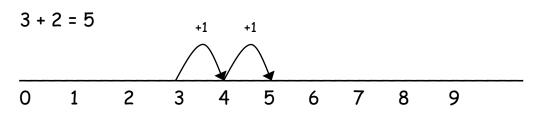
Pupils should be taught to:

- read, write and interpret mathematical statements involving addition (+) and equals (=) signs
- represent and use number bonds within 20
- add one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as $7 = \Box + 5$.

Children use number tracks, numberlines and practical resources, to support calculation.



They count on from the largest number.



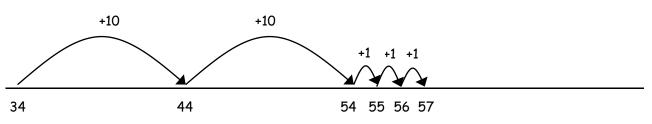
<u>YEAR 2</u>

Statutory requirements					
Pupils should be taught to:					
add numbers using concrete objects, pictorial representations, and mentally, including:					
 a two-digit number and ones 					
 a two-digit number and tens 					
 two two-digit numbers 					
 adding three one-digit numbers 					

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

First counting on in tens and ones.

34 + 23 = 57



Children then progress to more efficient methods by adding units in one jump and then tens in one jump.

When adding two digit numbers the children can use partitioning which can help with mental calculations too.

26 + 43 = 20 + 40 = 60 6 + 3 = 960 + 9 = 69

In Y2 the children are also introduced to the column addition method, which will also support their understanding of place value.

26 <u>43</u> + <u>69</u>

YEAR 3

Statutory requirements

Pupils should be taught to:

add numbers with up to three digits, using formal written methods of columnar addition

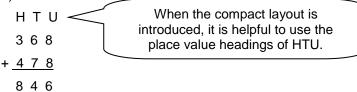
Build up starting with 2 digits + 2 digits

Introducing the compact layout

- no carrying (demonstration stage only), e.g. 54 + 35, 326 + 271
- carrying from units to tens, e.g. 47 + 26, 368 + 423
- carrying from tens to hundreds, e.g. 368 + 481
- carrying from units to tens and tens to hundreds, e.g. 47 + 76, 368 + 478
- a mixture of 'carries'

E.g. demonstrate the method children currently use alongside the new method. Children should then

practise the new method. (368 + 478)



When the compact layout is introduced, the language of place value should continue to be used but when children are confident, they will use 'digit-speak', e.g. for the addition of two three-digit numbers above, they are likely to say:

1 1

- 8 add 8 is sixteen; 6 in the answer and carry 1
- 6 add 7 is 13 plus the carry 1 is 14; 4 in the answer and carry 1
- 3 add 4 is 7 plus the carry 1 is 8
- Answer is 846

This is a form of shorthand that speeds up the process of addition. If children consistently carry out a range of calculations correctly, it is likely that they understand the process. However, teachers should occasionally check their understanding by asking children to explain what exactly they mean at each stage of the calculation, e.g. what does that 'carry 1' really mean?

Statutory requirements

Pupils should be taught to:

 add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate

See the column method shown for Y3

Children in Y4 will meet some fairly simple additions of £.p. some children may be able to use the compact column method, but those who have not learnt how to use this method should change pounds to pence and add using a compact method.

Ensure that children are aware that the decimal points should line up especially when using mixed amounts. Eg $\pm 3.59 + 78p =$

3.59 <u>0.78</u> + <u>4.37</u> 1 1

YEAR 5

Statutory requirements

Pupils should be taught to:

 add whole numbers with more than 4 digits, including using formal written methods (columnar addition)

Continue column addition method taught in earlier year groups.

These should be extended through addition of money and measures.

YEAR 6

There are no formal statutory requirements for addition in Y6. They are expected to continue with the formal written methods and apply this to problem solving.

Progression towards a written method for subtraction

EARLY STAGES (EYFS)

Children will engage in a variety of counting songs and rhymes and practical activities.

In practical activities and through discussion they will begin to use the vocabulary associated with subtraction.

They will find one less than a given number.

They will begin to relate subtraction to 'taking away' using objects to count 'how many

are left' after some have been taken away.

6 - 2 = 4



Take two apples away. How many are left?'

Children will begin to count back from a given number.

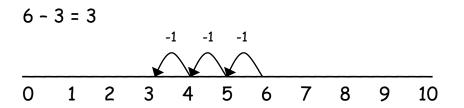
<u>YEAR 1</u>

Statutory requirements

Pupils should be taught to:

- read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs
- subtract one-digit and two-digit numbers to 20, including zero

The numberline should be used as the children will be using this for addition too.



The numberline should also be used to show that 6 - 3 means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.

Children should also be taught the counting on method, using a numberline; this will establish the relationship between the operations of addition and subtraction.

$$9 10 11$$

<u>YEAR 2</u>

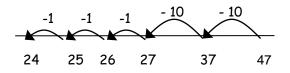
Statutory requirements

Pupils should be taught to:

- subtract numbers using concrete objects, pictorial representations, and mentally, including:
 - a two-digit number and ones
 - a two-digit number and tens
 - two two-digit numbers

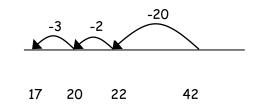
Children can continue to use a numberline counting back and counting on when finding small differences. First counting back in ones then progressing to tens and ones and then beyond.

47 - 23 = 24



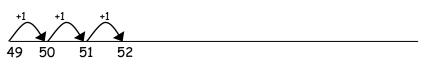
Bridging through ten can help children become more efficient.

42 - 25 = 17



Counting on to find a small difference

52 - 49 = 3



Children should be taught to use the column subtraction method without 'carrying'. This supports place value and prepares for formal written methods with larger numbers.

	TU —	
56 - 23 = 33	56-	When the formal layout is introduced,
	<u>23</u>	it is helpful to use the place value
	<u>33</u>	headings of TU.

Statutory requirements

Pupils should be taught to:

subtract numbers with up to three digits, using formal written methods of columnar subtraction

Children should be confident in carrying out subtraction by counting back or on, using a numberline, before being introduced to the formal method of decomposition. Some children find decomposition difficult.

Decomposition

Before the introduction of the formal written method for subtraction (decomposition), children should be able to:

- 1. recall all subtraction facts to 20;
- 2. subtract multiples of 10 (such as 160 70) using the related subtraction fact, 16 7, and their knowledge of place value;
- 3. partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

Teach the decomposition method in this order:-

- 1. TU TU, then HTU TU and HTU HTU, exchange from tens to units, e.g. 71 46, 173 38, 774 248
- 2. HTU HTU, exchange from hundreds to tens, e.g. 553 272
- 3. HTU HTU, exchange from tens to units and from hundreds to tens, e.g. 635 278

71 – 46 =

This should be formally recorded.

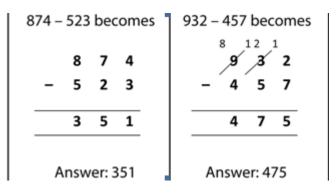
Statutory requirements

Pupils should be taught to:

 subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate

Countinue, or begin, with the formal written method of decomposition as set out in Y3. Extending this to

- Subtract numbers with up to four digits, including numbers with different numbers of digits
- Subtraction with numbers involving zeros e.g 5001 2345
- Subtract decimals, money and measures



<u>YEAR 5</u>

Statutory requirements

Pupils should be taught to:

 add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)

Follow column subtraction (decomposition) method taught in earlier year groups.

YEAR 6

There are no formal statutory requirements for subtraction in Y6. They are expected to continue with the formal written methods and apply this to problem solving.

Progression towards a written method for multiplication

EARLY STAGES (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving doubling.



'Three apples for you and three apples for me. How many apples altogether?'

YEAR 1

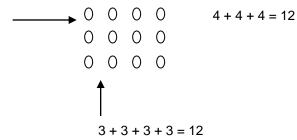
Statutory requirements

Pupils should be taught to:

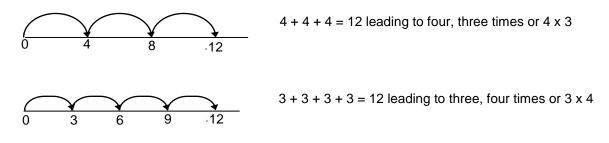
- solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Pictorial recording as children develop understanding of repeated addition, e.g. counting in sets of two, e.g. sets of two pence coins, five, e.g. tally bundles, tens, e.g. sets of 10 pennies.



• Use of arrays to illustrate repeated addition.



 Record repeated addition on a number line e.g. for the array above,



Using such models will help develop children's understanding of the commutativity of multiplication, e.g. $6 \times 2 = 2 \times 6$

<u>YEAR 2</u>

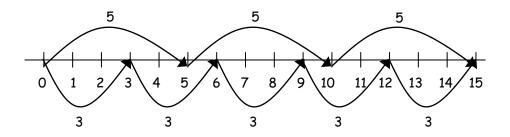
Statutory requirements

Pupils should be taught to:

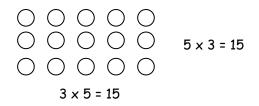
- recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (x) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative)

Children should continue with the numberline, to show repeated addition, and commutative.

Children should know that 3×5 has the same answer as 5×3 . This can also be shown on the number line.



Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



Year 3 to Year 6

Before the introduction of formal written methods for multiplication, children should be able to:

- recall multiplication facts for the tables used;
- partition numbers into multiples of one hundred, ten and one;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using the column method.

Statutory requirements

Pupils should be taught to:

- recall and use multiplication facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication using the multiplication tables that they
 know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal
 written methods

Year 3 use the grid method with tables children are confident with

Grid method

TU x U (Short multiplication – multiplication by a single digit)

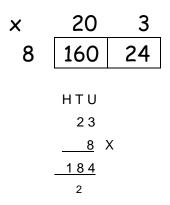
23 x 8

Children will approximate first 23×8 is approximately $25 \times 8 = 200$

X	20	3
8	160	24
	. <u></u>	

Short multiplication

• TU x U, e.g. 23 x 8 Demonstrate alongside the grid layout. Children use short multiplication.

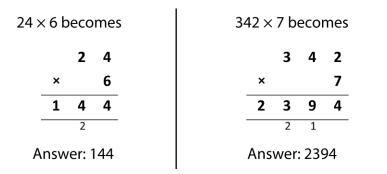


Statutory requirements

Pupils should be taught to:

• multiply two-digit and three-digit numbers by a one-digit number using formal written layout

Children should progress to using the formal written layout for multiplication problems.



This can also be extended to include money £.p x U, e.g. £3. 64 x 8

Note: Children who are continuing to use the grid layout should, when working with money or measures, convert to the smaller measure e.g. convert £.p to pounds, and then back to the original units if required.

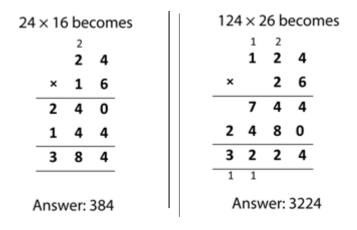
YEAR 5

Statutory requirements		
Pupils should be taught to:		
 multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers 		
 multiply whole numbers and those involving decimals by 10, 100 and 1000 		

Short Multiplication

24 × 6 becomes	342×7 becomes	2741 × 6 becomes		
2 4	3 4 2	2741		
× 6	× 7	× 6		
1 4 4	2 3 9 4	1 6 4 4 6		
2	2 1	4 2		
Answer: 144	Answer: 2394	Answer: 16 446		

Long Multiplication -placing of carrying figures to be determined by discussion



YEAR 6

Statutory requirements
Pupils should be taught to:
 multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method
of long multiplication

Children should apply the formal written methods taught in earlier year groups to more complex numbers, including decimals and measures.

Progression towards a written method for division

EARLY STAGES (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving halving and sharing.

Share the apples between two people.



'Half of the apples for you and half of the apples for me.'

YEAR 1

Pupils should be taught to:

- solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Use **sharing** to answer division questions such as:

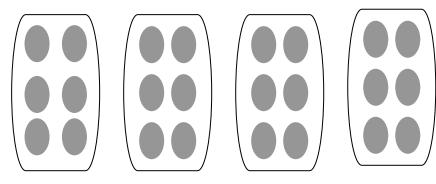
Share this bag of 12 counters into 3 pots.



• Experience division as grouping, such as:

24 eggs are packed in boxes of 6. How many boxes are needed?

Take 6 eggs and pack the first box. Continue until there are no eggs left. Then count how many boxes have been used, recording pictorially.



<u>YEAR 2</u>

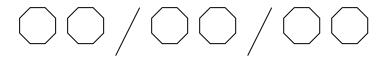
Statutory requirements

Pupils should be taught to:

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for division and write them using the division (÷) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot

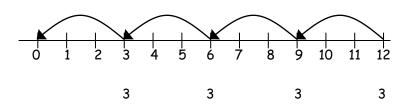
Introduce the number sentence and allow children to use sharing or grouping pictorial representations to solve.

There are 6 sweets, how many people can have 2 sweets each? $6 \div 2 = 3$



Repeated subtraction using a number line or bead bar

$12 \div 3 = 4$



Using symbols to stand for unknown numbers to complete equations using inverse operations

 $\Box \div 2 = 4 \qquad 20 \div \bigtriangleup = 4 \qquad \Box \div \bigtriangleup = 4$

Experience divisions that give rise to remainders, such as, in a sharing context:

Three friends share 17 marbles equally. How many marbles does each friend get? How many marbles are left over?

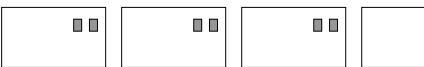


17 ÷ 3 =

They get 5 marbles each and there are 2 left over.

.. and in the context of grouping:

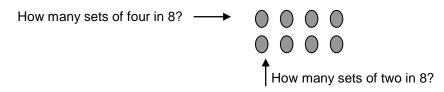
Grandma is sending Christmas cards. She needs 2 stamps to stick on each envelope. She has 11 stamps, how many cards can she send?





She can send 5 cards and has 1 stamp left over.

Relate division to multiplication using arrays.



Year 3 to Year 6

To progress towards short division, children need to be able to:

- understand and use the vocabulary of division; •
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts for the tables used; •
- recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using • their knowledge of division facts and place value;
- know how to find a remainder working mentally, e.g. find the remainder when 48 is divided by 5; •
- understand and use multiplication and division as inverse operations.

YEAR 3

Statutory requirements

Pupils should be taught to:

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for division using mental and progressing to formal written methods

Expanded layout for short division

Model this to support understanding the place value within the compact method but children are not expected to record this method. Record in compact form (short division)

Stage 1

TU \div U, no remainder and no carrying, e.g. 69 \div 3

$$20 \quad 3$$

$$3 \quad 60 \quad 9$$

$$69 \div 3 = 23$$

Stage 2

TU ÷ U, remainder but no carrying, e.g. 67 ÷ 3

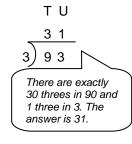
$$\underbrace{\begin{array}{c}
 20 & 2 \text{ R 1} \\
 3 & 60 & 7
 \end{array}}$$

67 ÷ 3 = 22 R 1

92 ÷ 4 = 23

Stage 3 (compact division)

TU \div U, no remainder and no carrying, e.g. 93 \div 3



YEAR 4

Statutory requirements

There are no formal statutory requirements linked to division in Year 4.

Children should practise and consolidate the division work in Year 3. Extended by introducing the compact short division method with remainders.

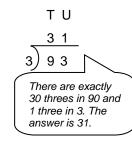
Compact layout for short division

Initially, children will continue to use correct place value vocabulary, but as they become more confident,

they can be introduced to 'digit-speak'. Dialogue with place value vocabulary is illustrated for Stages 1-4 and 'digit-speak' shown for Stage 4.

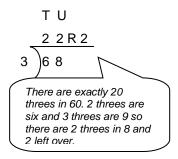
Stage 1

TU \div U, no remainder and no carrying, e.g. 93 \div 3



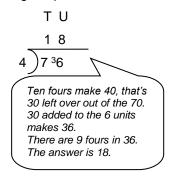
Stage 2

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TU ÷ U, remainder but no carrying, e.g. 68 ÷ 3
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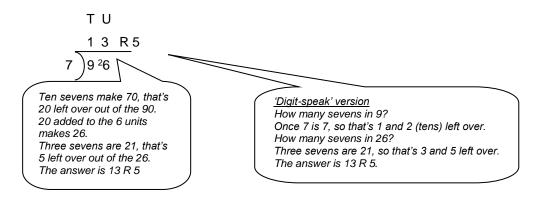
Stage 3

TU \div U, carrying from T to U but no remainder, e.g. 76 \div 4. When dealing with carrying figures, relate to knowledge of place value.



Stage 4

TU \div U with carrying and remainder, e.g. 96 \div 7



Statutory requirements

Pupils should be taught to:

 divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Continue to develop the short division method as follows:-

Extending the compact layout for short division

Order of difficulty of calculations:

- 1. No remainder, no carrying, e.g. 844 ÷ 4
- 2. Remainder, no carrying, e.g. 486 ÷ 4
- 3. No remainder, carrying from T to U, e.g. 860 ÷ 4
- 4. No remainder, carrying from H to T, e.g. 928 ÷ 4
- 5. No remainder, carrying from H to T and T to U, e.g. 984 ÷ 4
- 6. Remainder and carrying, e.g. 743 ÷ 4
- 7. Examples where consideration needs to be given to the placing of the quotient,

Initially pupils should cross out the hundreds digit and carry it over to the tens as well as placing zero in the quotient.

8. Examples where there are zeros in the quotient, e.g. $818 \div 4,5609 \div 8$

Emphasise zero as place holder.

9. Express remainders as fractions, e.g. 387 ÷ 4

$$4\frac{0\ 9\ 6}{3\ 8\ 27}^{3/4}$$

- 10. Decimals
- 11. Measures
- 12. Divide amounts of money,

e.g. Grandma emptied her money box. There was £12.46. She shared it equally between her five grandchildren. How much did they each get and how much was left over?

Each child got £2.49 and there was 1p left over.

13. Varied contexts where the remainder is expressed as a whole number, or a fraction, or a decimal or where the quotient needs to be rounded up (e.g. 72 children are going camping. Each tent holds 5 children. How many tents do they need?), or down (The farmer has collected 91 eggs. Egg boxes each hold 6 eggs. How many boxes can he fill?).

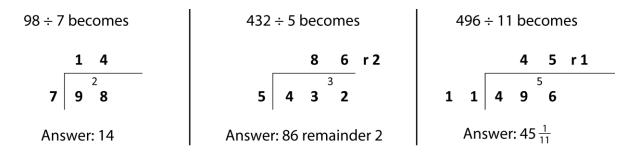
YEAR 6

Statutory requirements

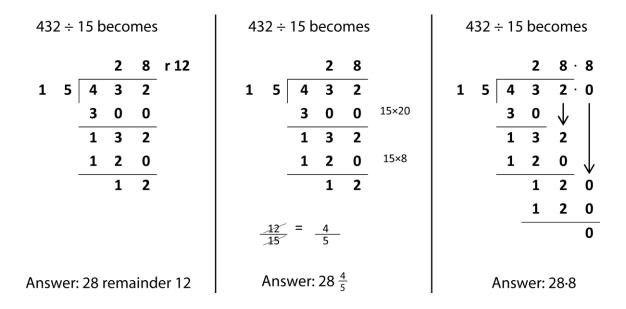
Pupils should be taught to:

- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

Short Division



Long Division



• Challenge with extended digits, decimal numbers, money and measures including remainders